

The spin structure function of the neutron

Anthony W. Thomas^{a*} ^bCSSM|Special Research Centre for the Subatomic Structure of Matter
and Department of Physics and Mathematical Physics,
The University of Adelaide, Adelaide SA 5005, Australia

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The neutron spin structure function, g_{1n} , has been of considerable interest recently in connection with the Bjorken sum rule and the proton spin crisis. Work on this problem has concentrated on measurements at low- x . We recall the important, non-perturbative physics to be learnt by going instead to larger values of x and especially from a determination of the place where the expected sign change occurs. Of course, in order to obtain neutron data one must use nuclear targets and apply appropriate corrections. In this regard, we review recent progress concerning the various nuclear corrections that must be applied to measurements on polarised ^3He .

1. INTRODUCTION

Since the original discovery of the “proton spin crisis” [1] there has been enormous progress in our knowledge and understanding of the spin structure functions of the nucleon [2]. It is well known that through the U(1) axial anomaly polarised gluons can contribute to the spin structure function of the proton [3,4] and that this is quite likely the source of most of the discrepancy with the naive Ellis-Jaffe sum rule. Tests of this idea are now focussed on direct measurements of the polarised gluon content at Hermes, COMPASS and RHIC.

In comparison with the Ellis-Jaffe sum rule, the Bjorken sum rule is still on a very sound theoretical foundation and provides a crucial test of our understanding of QCD. In order to check it one needs data for the neutron spin structure function, a task complicated by the absence of free neutron targets. Traditionally neutron data has been extracted from data on the deuteron [5–7] by subtracting the proton contribution. Apart from Fermi motion and binding corrections this is complicated by the presence of completely new terms not present for a free nucleon [8] – although

numerical estimates suggest that these terms are negligible for a system as weakly bound as the deuteron. The main problem with the deuteron is that the n and p spins are aligned so that one has to subtract two numbers of comparable size (g_{1p} from g_{1D}) to get the neutron data.

An attractive alternative is to use polarised ^3He [9,10], where the spins of the two protons are mainly coupled to zero, so that the neutron carries most of the nuclear spin. As a consequence, the corrections to be applied to the data to obtain the neutron structure function are expected to be much smaller than for the deuteron. Yet, binding and Fermi motion are larger, as are shadowing corrections and meson exchange currents – especially those involving the Δ . The shadowing corrections are especially significant in the small- x region, where most of the contribution from polarised gluons is also concentrated. In section 3 we outline recent progress in the calculation of those corrections which are relevant at intermediate and large- x . First, however, we recall the main non-perturbative aspects of nucleon structure that can be tested by extending measurements to larger values of x than have so far been accessible.

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2. LARGE- x AND THE NON-PERTURBATIVE STRUCTURE OF THE NUCLEON

While the emphasis for the last decade has been on extending our knowledge of spin structure functions to smaller x , we stress that there is a lot to be learnt by heading in the opposite direction. Large- x has proven to be a source of surprises for unpolarised structure functions, with recent analysis [11] suggesting that the d/u ratio may not go to zero (unlike the standard parametrisations of parton distributions universally assume) but is rather consistent with the predictions of perturbative QCD (pQCD) [12]. Further experiments are planned at places like JLab to test this analysis [13].

In the spin dependent case we have no idea whatsoever of the behaviour of g_{1n} beyond $x \sim 0.4$. On the basis of both pQCD and SU(6) [14], one expects A_{1n} to approach 1 as $x \rightarrow 1$ [15,16]. It is vital to test this prediction. If it fails we understand nothing about the valence spin structure of the nucleon! Looking in a little more detail, we see that the present data at small- x corresponds to a negative asymmetry and hence there must be a crossover at some intermediate x value. Locating the crossover is an important experimental challenge. From the theoretical point of view the value of x at which this occurs is the result of a competition between the SU(6) valence structure [14] and the chiral corrections [17,18].

The positive sign of the neutron asymmetry in the larger x region is a result of the dominance there of $S = 0$ spectator pairs (which have lower mass than $S = 1$ pairs) [14]. Within the SU(6) framework, the only valence quark with a spin-0 pair of spectators is a d -quark which has its spin aligned with the spin of the neutron – and hence the asymmetry is positive. This phenomenon has been studied at great length, not only for the spin-flavor dependence of nucleon parton distributions [19] but also for the parton distributions and fragmentation functions of other members of the nucleon octet [20,21]. The competition comes from the $N\pi$ and $\Delta\pi$ Fock components of the wave function of the nucleon [18], corresponding to the leading and next-to-leading non-analytic

chiral behaviour of the structure function [22]. In view of the interest in di-quark models of the nucleon [23], as well as the role of dynamical symmetry breaking, it is imperative to have as much experimental guidance as possible and the insight from accurate data on g_{1n} at intermediate and large x would be extremely valuable.

3. EXTRACTING G_{1n} FROM ^3He DATA

Within the usual impulse approximation, $g_1^{^3\text{He}}$ can be represented as the convolution of the neutron (g_1^n) and proton (g_1^p) spin structure functions with the spin-dependent nucleon light-cone momentum distributions $\Delta f_{N/^3\text{He}}(y)$, where y is the ratio of the struck nucleon to nucleus light-cone plus components of the momenta

$$g_1^{^3\text{He}}(x, Q^2) = \int_x^3 \frac{dy}{y} \Delta f_{n/^3\text{He}}(y) g_1^n(x/y, Q^2) + \int_x^3 \frac{dy}{y} \Delta f_{p/^3\text{He}}(y) g_1^p(x/y, Q^2). \quad (1)$$

The motion of the nucleons inside the nucleus (Fermi motion) and their binding are parametrized through the distributions $\Delta f_{N/^3\text{He}}$, which can be readily calculated using the ground-state wave function of ^3He . Detailed calculations [24–26] by various groups, using different ground-state wave functions for ^3He , have come to similar conclusions, namely that $\Delta f_{N/^3\text{He}}(y)$ is sharply peaked around $y \approx 1$ because of the small average separation energy per nucleon. Thus, Eq. (1) is often approximated by

$$g_1^{^3\text{He}}(x, Q^2) = P_n g_1^n(x, Q^2) + 2P_p g_1^p(x, Q^2), \quad (2)$$

where P_n (P_p) are the effective polarizations of the neutron (proton) inside polarized ^3He , defined by

$$P_{n,p} = \int_0^3 dy \Delta f_{n,p/^3\text{He}}(y). \quad (3)$$

In the first approximation to the ground-state wave function of ^3He , only the neutron is polarized. In this case, $P_n=1$ and $P_p=0$. Realistic approaches to the wave function of ^3He include also higher partial waves, notably the D and S' partial waves. This leads to the depolarization of the spin of the neutron and polarization

of protons in ^3He . The average of calculations with several models of the nucleon-nucleon interaction and three-nucleon forces can be summarized as $P_n = 0.86 \pm 0.02$ and $P_p = -0.028 \pm 0.004$ [27]. The calculations of [26] give similar values: $P_n = 0.879$ and $P_p = -0.021$ for the PEST potential with 5 channels. We shall use these values for P_n and P_p throughout this paper. Most of the uncertainty in the values for P_n and P_p comes from the uncertainty in the D -wave of the ^3He wave function. Thus, for the observables that are especially sensitive to the poorly constrained P_p , any theoretical predictions carry a significant uncertainty. One example of such an observable is the point where the neutron structure function g_1^n has a node.

4. ROLE OF THE $\Delta(1232)$

The description of the nucleus as a mere collection of protons and neutrons is incomplete. In polarized DIS on the tri-nucleon system, this observation can be illustrated by the following example [28]. The Bjorken sum rule relates the difference of the first moments of the proton and neutron spin structure functions to the axial vector coupling constant of the neutron, $g_A = 1.2670 \pm 0.0035$ [2],

$$\int_0^1 (g_1^p(x, Q^2) - g_1^n(x, Q^2)) dx = \frac{1}{6} g_A \left(1 + O\left(\frac{\alpha_s}{\pi}\right)\right). \quad (4)$$

Here the QCD radiative corrections are denoted as “ $O(\alpha_s/\pi)$ ”. This sum rule can be straightforwardly generalized to the ^3He - ^3H system, with $g_A|_{\text{triton}}$, the axial vector coupling constant of the triton ($g_A|_{\text{triton}} = 1.211 \pm 0.002$ [29]) replacing g_A . Taking the ratio of the Bjorken sum rules for $A = 3$ and the nucleon, one obtains

$$\frac{\int_0^3 (g_1^{^3\text{H}}(x, Q^2) - g_1^{^3\text{He}}(x, Q^2)) dx}{\int_0^1 (g_1^p(x, Q^2) - g_1^n(x, Q^2)) dx} = \frac{g_A|_{\text{triton}}}{g_A} = 0.956 \pm 0.004. \quad (5)$$

Note that the QCD radiative corrections cancel exactly in Eq. (5).

Assuming charge symmetry between the ^3He and ^3H ground-state wave functions, one can

write the triton spin structure function $g_1(x, Q^2)$ in the form:

$$g_1^{^3\text{H}}(x, Q^2) = \int_x^3 \frac{dy}{y} \Delta f_{n/^3\text{He}}(y) \tilde{g}_1^p(x/y, Q^2) + \int_x^3 \frac{dy}{y} \Delta f_{p/^3\text{He}}(y) \tilde{g}_1^n(x/y, Q^2). \quad (6)$$

This leads to the following estimate for the ratio of the nuclear to nucleon Bjorken sum rules

$$\frac{\int_0^3 (g_1^{^3\text{H}}(x, Q^2) - g_1^{^3\text{He}}(x, Q^2)) dx}{\int_0^1 (g_1^p(x, Q^2) - g_1^n(x, Q^2)) dx} = (P_n - 2P_p) \frac{\tilde{\Gamma}_p - \tilde{\Gamma}_n}{\Gamma_p - \Gamma_n} = 0.921 \frac{\tilde{\Gamma}_p - \tilde{\Gamma}_n}{\Gamma_p - \Gamma_n}. \quad (7)$$

Here we used $P_n = 0.879$ and $P_p = -0.021$, and $\tilde{\Gamma}_N = \int_0^1 dx \tilde{g}_1^N(x)$ and $\Gamma_N = \int_0^1 dx g_1^N(x)$ are, respectively, the spin sums for bound and free nucleons. Such off-shell corrections were estimated within the framework of the quark-meson coupling model [30] in Ref. [18]. They were not large and showed a tendency to decrease, rather than increase, the bound nucleon spin structure functions (i.e. $(\tilde{\Gamma}_p - \tilde{\Gamma}_n)/(\Gamma_p - \Gamma_n) < 1$). Thus, one can immediately see that the theoretical prediction for the ratio of the Bjorken sum rule for the $A = 3$ and $A = 1$ systems (Eq. (7)), based solely on nucleonic degrees of freedom, underestimates the experimental result for the same ratio (Eq. (5)) by about 3.5%. This demonstrates the need for new nuclear effects.

It has been known for a long time that non-nucleonic degrees of freedom, such as pions, vector mesons, the $\Delta(1232)$ isobar, can play an important role in the calculation of some low-energy observables in nuclear physics. In particular, the analysis of Ref. [31] demonstrated that the two-body exchange currents involving a Δ -isobar increase the theoretical prediction for the axial vector coupling constant of the triton by about 4%, which makes it consistent with experiment. In order to preserve the Bjorken sum rule, exactly the same mechanism must be present in case of deep inelastic scattering on polarized ^3He and ^3H . Indeed, as explained in Refs. [28,34], the direct correspondence between the calculations of the

Gamow-Teller matrix element in the triton β decay and the Feynman diagrams for DIS on ^3He and ^3H (see Fig. 1 of [34]) requires that two-body exchange currents should play an equal role in both processes. As a result, the presence of the Δ in the ^3He and ^3H wave functions should increase the ratio of Eq. (7) and make it consistent with Eq. (5).

The contribution of the $\Delta(1232)$ to $g_1^{^3\text{He}}$ is realized through diagrams involving the non-diagonal interference transitions $n \rightarrow \Delta^0$ and $p \rightarrow \Delta^+$. This requires new spin structure functions, $g_1^{n \rightarrow \Delta^0}$ and $g_1^{p \rightarrow \Delta^+}$, as well as the effective polarizations $P_{n \rightarrow \Delta^0}$ and $P_{p \rightarrow \Delta^+}$. Taking into account the interference transitions leads to a correction to the $A = 3$ spin structure functions, δg_1 :

$$\delta g_1 = \pm \left[2P_{n \rightarrow \Delta^0} g_1^{n \rightarrow \Delta^0} + 4P_{p \rightarrow \Delta^+} g_1^{p \rightarrow \Delta^+} \right], \quad (8)$$

where the \pm signs correspond to $^3\text{He}/\text{H}$, respectively.

The interference structure functions can be related to g_1^p and g_1^n within the quark parton model using the general structure of the SU(6) wave functions [14,21]

$$g_1^{n \rightarrow \Delta^0} = g_1^{p \rightarrow \Delta^+} = \frac{2\sqrt{2}}{5} (g_1^p - 4g_1^n). \quad (9)$$

This simple relationship is valid in the range of x and Q^2 where the contribution of sea quarks and gluons to g_1^N can be safely omitted, i.e. for $0.5 \leq Q^2 \leq 5 \text{ GeV}^2$ and $0.2 \leq x \leq 0.8$ if the parametrization of Ref. [32] is used.

In principle, the effective polarizations of the interference contributions $P_{n \rightarrow \Delta^0}$ and $P_{p \rightarrow \Delta^+}$ can be calculated using a ^3He wave function that includes the Δ resonance. This is an involved computational problem. Instead, we chose to find $P_{n \rightarrow \Delta^0}$ and $P_{p \rightarrow \Delta^+}$ by *requiring* that the use of the ^3He and ^3H structure functions of Eq. (8) gives the experimental ratio of the nuclear to nucleon Bjorken sum rules (5). This yields the effective polarizations [33]:

$$2(P_{n \rightarrow \Delta^0} + 2P_{p \rightarrow \Delta^+}) = -0.025. \quad (10)$$

(Note that this value is very close to that reported in Ref. [34].)

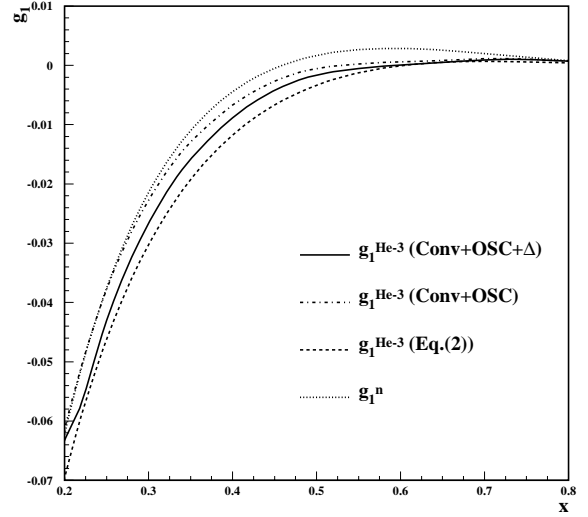


Figure 1. The full spin structure function, $g_1^{^3\text{He}}$ (solid curve), compared with that computed by allowing for Fermi motion and binding, with (dash-dotted curve) and without (dashed curve) the inclusion of off-shell corrections (OSC), estimated in the QMC model. The free neutron spin structure function, g_1^n , is shown by the dotted curve. For all curves $Q^2 = 4 \text{ GeV}^2$.

Clearly one can now write an explicit expression for the ^3He spin structure function, which takes into account the additional diagrams corresponding to the non-diagonal interference $n \rightarrow \Delta^0$ and $p \rightarrow \Delta^+$ transitions; thus ensuring agreement with the experimental value of the ratio of the Bjorken sum rules. The results of such a calculation of $g_1^{^3\text{He}}$ at $Q^2 = 4 \text{ GeV}^2$ are presented in Fig. 1 as the solid curve. One can see from Fig. 1 that the presence of the $\Delta(1232)$ isobar in the ^3He wave function works to decrease $g_1^{^3\text{He}}$ relative to the prediction of Fermi motion and binding alone. This decrease is 12% at $x = 0.2$ and increases at larger x , peaking for $x \approx 0.46$, where g_1^n changes sign.

We note that, since the convolution formalism implies *incoherent* scattering off nucleons and nucleon resonances of the target, coherent nuclear

effects present at small values of Bjorken x are being ignored here. In Ref. [34], the role played by two coherent effects, namely nuclear shadowing and antishadowing, in DIS on polarized ^3He were also considered. As these corrections are not significant in the large- x region, which is of concern to us here, we pursue this question no further.

4.1. Correction to the neutron asymmetry at large- x

The DIS asymmetry A_1^T for any target T is proportional to the spin structure function g_1^T :

$$g_1^T = \frac{F_2^T}{2x(1+R)} A_1^T, \quad (11)$$

where $R = (F_2^T - 2xF_1^T)/(2xF_1^T)$ and $F_{1,2}^T$ are inclusive spin-averaged structure functions. It is assumed in Eq. (11) that the transverse spin asymmetry, A_2^n , is negligibly small and that R does not depend on the choice of target.

Applying this definition of A_1^T to the ^3He , proton and neutron targets and including the effects of Fermi motion, binding and Δ -isobars outlined above, one obtains for the neutron asymmetry A_1^n

$$A_1^n = \frac{F_2^{^3\text{He}}}{P_n F_2^n (1 + \frac{0.056}{P_n})} \left(A_1^{^3\text{He}} - 2 \frac{F_2^p}{F_2^{^3\text{He}}} P_p A_1^p \left(1 - \frac{0.014}{2P_p} \right) \right). \quad (12)$$

Provided that the proton asymmetry, A_1^p , is constrained well by the experimental data, the largest theoretical uncertainty comes from the uncertainty in the proton spin polarization P_p . We estimate that the uncertainty in the second term of Eq. (12) and, thus, in the position of the point where A_1^n has a zero, is of the order 10% [33].

In Eq.(12) the terms proportional to 0.056 and 0.014 represent the correction to A_1^n associated with the Δ isobar. Both terms are important for the correct determination of A_1^n . The term proportional to 0.056 decreases the absolute value of A_1^n by about 6%. Moreover, if $A_1^{^3\text{He}}$ is negative, the second term proportional to 0.014 would work in the same direction of decreasing of $|A_1^{^3\text{He}}|$. Since the term proportional to 0.014 is always positive, this means that the true A_1^n should turn

positive at lower values of x compared to the situation when the effect of the Δ is ignored. It is therefore vital to account for these corrections in any extraction of g_{1n} from ^3He data in this region [35].

5. CONCLUSION

In conclusion we reiterate the importance of experiments aimed at extracting information on the large- x behaviour of the nucleon spin structure – especially that of the neutron. The results cast important light onto the valence structure of the nucleon, testing our understanding of SU(6) symmetry and the applicability of di-quark models. In order to extract this information from nuclear targets it is important to use a thorough theoretical analysis, including in particular the effects of Δ -isobars in the three-body wave function.

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